Magnetic Field Evaluation at Vertex by Boundary Integral Equation Derived from Scalar Potential of Double Layer Charge

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Abstract — Adopting an integral representation of the scalar potential due to the double layer charge σ_d, we derive a boundary integral equation (BIE) with one unknown to solve magnetostatic problems. Since σ_d produces only the potential gap without disturbing the normal magnetic flux density, the field is accurately formulated even with one unknown. The BIE is capable of treating robustly geometrical singularities at edges and corners. In this paper, we study how to evaluate the field at a vertex such as sharp edges and corners.

I. Introduction

When we adopt the volume integral equation approach [1] so as to formulate the scalar potential φ_B that gives the magnetic flux density $\boldsymbol{B}$ as $\boldsymbol{B}$=-grad(φ_B), we get a boundary integral equation (BIE) with the double layer charge σ_d as the state variable [2]. It seems there are several advantages over the conventional BIEs [3], [4] derived from the scalar potential φ_H, that gives the magnetic field $\boldsymbol{H}$ as $\boldsymbol{H}$=-grad(φ_H), but only a few papers have been reported. Enforcing the boundary condition of the continuity of the tangential component of $\boldsymbol{H}$, we derive a BIE with one unknown σ_d. Scalar potential formulation is computationally attractive but has fatal drawbacks due to the multi-valued function of exciting potential by current sources. By introducing a fictitious loop current to represent an all-purpose exciting potential, the BIE becomes applicable to generic problems. This paper presents how to utilize the BIE derived for evaluating $\boldsymbol{B}$ at the vertex.

The double layer charge σ_d is equivalent to φ_B that is definite and smoothly distributed even if the fields become infinite. Once σ_d is determined, $\boldsymbol{B}$ is evaluated with the help of Biot-Savart's law because σ_d is also equivalent to the loop current [3]. Therefore, it is expected that σ_d is capable of evaluating $\boldsymbol{B}$ at a vertex such as sharp edges and corners, where $\boldsymbol{B}$ approaches occasionally infinity.

II. Formulation of Magnetostatic Field

In the volume integral equation approach, a magnetic material with the surface S and volume V is replaced by fictitious current and charge [1]. The potential φ_M at an observation point P_o due to the magnetization $\boldsymbol{M}$ is given as

$$\varphi_{MP}=\int_V \frac{\boldsymbol{M}\cdot\boldsymbol{r}}{4\pi r^3}dV=\int_S \frac{\boldsymbol{M}\cdot d\boldsymbol{S}}{4\pi r}-\int_V \frac{\nabla\cdot\boldsymbol{M}}{4\pi r}dV \\ =\int_S \frac{M_s}{4\pi r}dS+\int_V \frac{m_v}{4\pi r}dV \tag{1}$$

where the subscript P denotes the value at P_o, r is the distance from an integral point P_i to P_o, $\boldsymbol{M}$ is defined as $\boldsymbol{M}=\boldsymbol{B}-\mu_0\boldsymbol{H}$ with the magnetic filed $\boldsymbol{H}$ and flux density $\boldsymbol{B}$ and the magnetic permeability μ_0 of free space, and m_v and M_S are called the volume and surface charges defined as $m_v=-\nabla\cdot\boldsymbol{M}$, $M_s=\boldsymbol{M}\cdot\boldsymbol{n}$ with the unit outward normal $\boldsymbol{n}$. Here, we assume that the magnetization property is linear, that is $\nabla\cdot\boldsymbol{M}=0$, and derive a BIE to evaluate $\boldsymbol{B}$ at vertices. The concept of magnetic shell [3], which is composed of the double layer charges σ_d, suggests that φ_{Ms} could be replaced by the potential due to σ_d. Taking this concept into account and assuming m_v=0, we get the total potential φ_B as

$$\varphi_{BP}=\varphi_{BeP}+\int_S \sigma_d \frac{\boldsymbol{n}_s\cdot\boldsymbol{r}}{4\pi r^3}dS \tag{2}$$

where φ_{Be} is the potentials at P_o produced by the exciting source, $\boldsymbol{r}$ is the distance from an integral point P_i on S to P_o and $\boldsymbol{n}_s$ is the unit outward normal at P_i.

The potentials on the surfaces with the subscripts o and i denoting the outer and inner sides are given as

$$\varphi_{BoP}=\varphi_{BeP}+\frac{\Omega_P}{4\pi}\sigma_{dP}+\int_S \sigma_d \frac{\boldsymbol{n}_s\cdot\boldsymbol{r}}{4\pi r^3}dS, \tag{3}$$

$$\varphi_{BiP}=\varphi_{BeP}-\left(1-\frac{\Omega_P}{4\pi}\right)\sigma_{dP}+\int_S \sigma_d \frac{\boldsymbol{n}_s\cdot\boldsymbol{r}}{4\pi r^3}dS \tag{4}$$

where Ω is the solid angle subtended at the singular point P_o on the surface S [5]. Applying the boundary condition of the continuous condition of the tangential magnetic field to (3) and (4), we derive a BIE as

$$\frac{\Omega_P\mu_r-\Omega_P+4\pi}{4\pi(\mu_r-1)}\sigma_{dP}+\int_S \sigma_d \frac{\boldsymbol{n}_s\cdot\boldsymbol{r}}{4\pi r^3}dS=-\varphi_{BeP} \tag{5}$$

where μ_r is defined as $\mu_r=\mu/\mu_0=|\boldsymbol{B}_i|/|\mu_0\boldsymbol{H}_i|$ and φ_{Be} is given by the potential φ_{Bc} due to the coil current I_c and the potential φ_{Bf} due to the fictitious loop current I_f (usually I_f=I_c) as

$$\varphi_{BeP}=\varphi_{BcP}+\varphi_{BfP}=\left(\mu_0\Omega_{cP}I_c+(\mu-\mu_0)\Omega_{fP}I_f\right)/(4\pi) \tag{6}$$

with the solid angle Ω_c subtended at P_o by the surface S_c surrounded by I_c and Ω_f subtended by the cross section S_f of magnetic core cut by S_c. If S_c doesn't cut the core, Ω_f=0.

III. Evaluation of Magnetic Flux Density

Once the distribution of σ_d has been obtained, $\boldsymbol{B}$ is evaluated with the help of Biot-Savart's law because σ_d is equivalent to the loop currents J_l. The surface element is divided further into the sub-elements as shown in Fig. 1, where the solid lines are for the original surface element and the broken lines are for the sub-element to evaluate $\boldsymbol{B}$. The calculating points P_o for obtaining σ_{d1}−σ_{d4} have been set at the point shown by ○. We set points P_e shown by ♦ at the center of the sub-elements surrounded by the solid and dotted lines and σ_{de} at P_e is interpolated with these σ_d as

$$\sigma_{de} = \sum_{i=1}^{4} N_i \sigma_{di} \tag{7}$$

where the shape function N_i is given as

$$N_1 = \frac{1}{4ab}(a-x)(b-y),\ N_2 = \frac{1}{4ab}(a+x)(b-y),$$
$$N_3 = \frac{1}{4ab}(a+x)(b+y),\ N_4 = \frac{1}{4ab}(a-x)(b+y)$$

with the local coordinates x and y at P_e and the side lengths a and b of the surface element. Then the loop current is introduced so as to circulate anticlockwise along the contour of the sub-element as shown by the arrows in the figure. Employing these loop currents, the total magnetic flux density $\boldsymbol{B}$ at P_o is given as follows.

$$\boldsymbol{B}_P = \boldsymbol{B}_{\sigma P} + \boldsymbol{B}_{eP} = \sum_{i=1}^{N_J} J_{li} \oint_{\Delta L_i} \frac{\boldsymbol{u}_J \times \boldsymbol{r}_L}{4\pi r_L^{\,3}} dL_i + \boldsymbol{B}_{eP} \tag{8}$$

where N_J is the total number of J_l that equals to the number of σ_d, the $\boldsymbol{u}_J$ is the direction of J_l, which circulates anticlockwise along the contour of σ_d, ΔL is the length of J_l, and $\boldsymbol{r}_L$ is the distance from P_i along J_l to P_o.

Eq. (8) is applicable to evaluate $\boldsymbol{B}$ at any point except on the surface. Even on the surface, (8) is capable of evaluating the normal component B_n by setting the calculating point P_e at the center of the sub-element as shown in Fig.1, but incapable of doing the tangential component $\boldsymbol{B}_t$. Since there is a gap $\Delta \boldsymbol{B}_t$ between the inner and outer $\boldsymbol{B}_t$, we take the $\Delta \boldsymbol{B}_t$ into account and get $\boldsymbol{B}_t$ as

$$\boldsymbol{n}_P \times \boldsymbol{B}_{oP} = \frac{2\mu_0}{\mu + \mu_0} \boldsymbol{n}_P \times \boldsymbol{B}_{sP},\ \boldsymbol{n}_P \times \boldsymbol{B}_{iP} = \frac{2\mu}{\mu + \mu_0} \boldsymbol{n}_P \times \boldsymbol{B}_{sP} \tag{9}$$

where $\boldsymbol{B}_s$ is $\boldsymbol{B}$ on the surface and evaluated by (8). When we evaluate $\boldsymbol{B}$ at a vertex, we set one loop current J_l on the surface elements, where one of the element corners joins at the vertex indicated by ● as shown in Fig. 1.

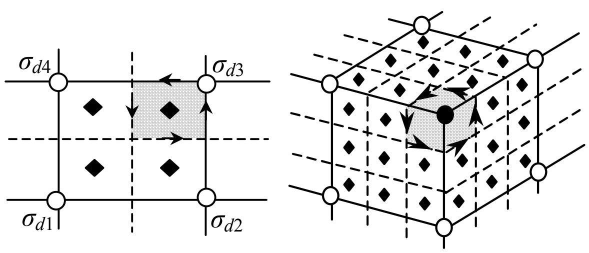

Fig.1 Subdivisions for evaluating $\boldsymbol{B}$ at ♦ on flat surface and ● at vertex.

IV. NUMERICAL VALIDATION OF PROPOSED APPROACH

We shall solve a magnetostatic problem with a cubic magnetic block (10x10x10 cm^3) placed in the uniform magnetic field $\boldsymbol{H}_{eo}$ of 1 T. The relative permeability μ_r is 1000, and the direction of $\boldsymbol{H}_{eo}$ is perpendicular to the block surface. The z-axis is set parallel to $\boldsymbol{H}_{eo}$ and the origin of axes is at a corner of the bottom surface of the block. The material surface is divided equally into N_e=6N^2. Employing the linear surface element in discretization of (5), we obtain σ_d at the node point shown by ○ and ● in Fig.1, and evaluate $\boldsymbol{B}$ by using (8). Fig.2 shows the computed results of $\boldsymbol{B}$ near the edge at x=0.2 cm and y=0.2 cm along the z-axis. The subscripts x, y and z, attached to B denote the components of $\boldsymbol{B}$, and B_x=B_y. As approaching the corners, $\boldsymbol{B}$ evaluated by the proposed method increases sharply.

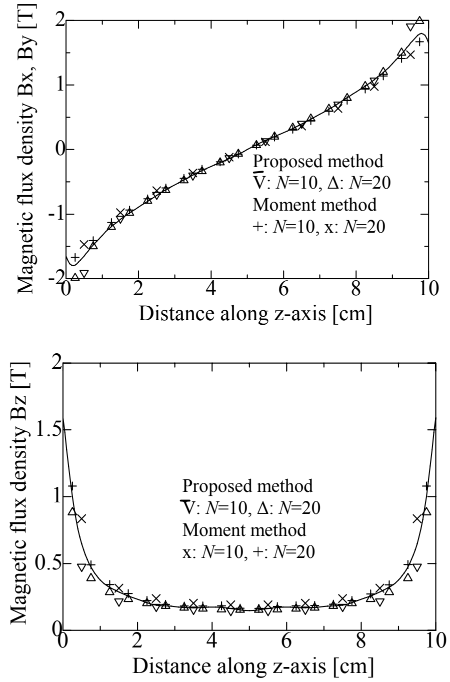

Fig.2 Computed results of magnetic flux density along z-axis at x, y=0.2 cm. The solid lines denote the results by the magnetic moment method [6] with meshes fine enough.

V. CONCLUSIONS

We have derived a BIE with the double layer charge as the state variable. Even if the BIE contains only one unknown, the solution fulfills completely the boundary condition and is expected to be accurate. The solution of the BIE gives directly the potential distribution. The potential is equivalent to the double layer charge and also the loop current. Therefore, once the double layer charges have been obtained, they give directly the magnetic flux density by virtue of Biot-Savart's law. We have evaluated the magnetic flux density near the edge of a magnetic block and confirmed that the evaluation is adequate. It is expected that the proposed method is capable of evaluating easily the fields at any points including the edge and corner.

VI. REFERENCES

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